**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer: - Mean(µ)= 45 minutes

Standard Deviation(σ) = 8 Minutes

Because the car is dropped off 10 minutes before work begins, the remaining time for servicing is 60 minutes - 10 minutes = 50 minutes.

Z-score Z = (X - μ) / σ X = remaining time available = 50 minutes Z = (50 - 45) / 8 = 0.625

probability associated with the Z-score of 0.625 using a conventional normal distribution table or statistical software. Assume that P (Z > 0.625) = 0.2676.

As a result, **the correct answer is B. 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer: - A. More employees at the processing center are older than 44 than between 38 and 44.

**False**, with a mean of 38, the distribution is typical, indicating that the majority of employees are roughly that age. There would be more workers aged 38 to 44 (within one standard deviation of the mean) than those over 44 (beyond one standard deviation from the mean), as a result.

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**False**, must take into account the proportion of workers who are 30 years of age or younger and within one standard deviation of the mean age (38 - 6 = 32). We are unable to determine the precise percentage, but we will guess that there are 36 employees who are under the age of 30.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer: -

Distributions:

2X1: 2X1's distribution is an enlarged version of X1's distribution. The distribution will still be normal, but with a mean of 2 (twice X1's mean) and a variance of 4 (four times X1's variance).

X1 + X2: X1 and X2 added together will also have a normal distribution. The total will have a mean of 2 (the sum of the means of X1 and X2) and a variance of 2 (the sum of the variances of X1 and X2) because X1 and X2 are independently and identically distributed (iid).

Parameters:

X1: The mean of X1 is 1, while the mean of 2X1 is 2, which is twice that. Four times the volatility of X1 is the variance of 2X1, which has a variance of 42.

X1 + X2: The average of X1 + X2 is equal to 2, which is the sum of X1 and X2's means. The sum of the variances of X1 and X2 is 22, which is the variance of X1 + X2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer:- Given:

Mean (μ) = 100

Standard deviation (σ) = √400 ≈ 20

A cumulative probability of 0.99 corresponds to a z-score of roughly 2.33.

Now, we can use the formula to determine the values of a and b:

a = μ - z \* σ

b = μ + z \* σ

Making the following substitutions:

a = 100 - 2.33 \* 20 ≈ 100 - 46.6 ≈ 53.4

b = 100 + 2.33 \* 20 ≈ 100 + 46.6 ≈ 146.6

As a result, 53.4 and 146.6, respectively, are the values of a and b that are symmetric about the mean and at which the chance of the random variable taking a value between them is 0.99.

The right response is **D. 48.5, 151.5**.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Answer: -

A. The profit distributions must be converted from dollars to rupees using the current exchange rate of $1 = 45.

Given:

Profit1 ~ N (5, 9) (in $ Million)

Profit2 ~ N (7, 16) (in $ Million)

Exchange rate: $1 = Rs. 45

multiply the mean and standard deviation of each distribution by the conversion rate in order to convert the profit distributions to Rupees.

Profit1 in Rupees: N (5 \* 45, 9 \* 45^2) = N (225, 182250) (in Rs. Million)

Profit2 in Rupees: N (7 \* 45, 16 \* 45^2) = N (315, 129600) (in Rs. Million)

1. must locate the associated z-scores for a 95% confidence interval in order to designate a rupee range based on the mean that contains 95% likelihood for the company's annual profit. A 95% confidence interval has a z-score of roughly 1.96.

For Profit1:

Rupee range: (225 - 1.96 \* √182250, 225 + 1.96 \* √182250) ≈ (141.05, 308.95) (in Rs. Million)

For Profit2:

Rupee range: (315 - 1.96 \* √129600, 315 + 1.96 \* √129600) ≈ (226.68, 403.32) (in Rs. Million)

1. compare the means of the profit distributions in Rupees to identify which division has a higher likelihood of posting a loss in any given year.

Profit1 mean in Rupees: 225 (in Rs. Million)

Profit2 mean in Rupees: 315 (in Rs. Million)

Profit1 is more likely to experience a loss in a given year because it has a smaller mean than Profit2, which has a larger mean.

The following are the questions' answers:

A. The range in rupees is (141.05, 308.95).

B. 187.79 is the 5th percentile of profit in rupees.

C. Profit1 division is more likely to experience a loss in any given year.